They were all sitting around one table. Mathematics faculty from two middle schools, two high schools, a community college and the local university. As they talked, a startling fact suddenly hit them: every one of their institutions offered the exact same course.

Sure, at one level, that course—Algebra 1—was taught to the most advanced students, while at another level it was taught to the least advanced students. But it still demonstrates how much these two systems have in common (not to mention how much overlap we tolerate).

And frankly, this is by no means the only such example. Consider the following:

- The fastest growing part of the high school curriculum during the 1980s and 1990s was in Advanced Placement, or other college-level courses;
- Over the same time period, the fastest growing part of the college mathematics curriculum was in remedial, or high school-level courses.
- More than 90% of mathematics enrollments in higher education are in courses also taught in high school.

A decade ago, the nation established a set of education goals that included specific targets for raising mathematics achievement by the year 2000. At decade’s end, America’s math students would then be first in the world.

The year 2000 has come and gone. The number of BAs awarded in mathematics declined by 20%, worsening an already critical shortage of qualified math teachers. And while our K-12 students know more mathematics now than they did in 1990, so do their peers in other countries. We did not improve enough to break out of our mediocre position in international comparisons.

You don’t have to look at the research for very long to see that falling short in one area relates to failure in the others. The short supply of mathematically proficient teachers hampers our
efforts to dramatically raise student achievement, which in turn, produces fewer college students interested in entering math fields, leading to a shorter supply of math majors, especially math majors who want to become teachers, and so on and on.

There is no clearer, more compelling case for collaborative K-16 action than this cycle of teaching and learning in mathematics.

In this issue of Thinking K-16, we take a close look at mathematics achievement and attainment in America in K-12 and in higher education. We also explore the extent to which we provide students with the opportunity to learn mathematics.

In Part Two, we focus on the mathematics curriculum in American schools and what needs to be improved. We also look at inequities in college prep math enrollments.

Part Three examines the distribution of math teachers. This section features analyses of brand new data from the national Schools and Staffing Survey, and shows that we still have a great deal of work to do to make sure that all students are taught by fully qualified math teachers.

Finally we ask where all the math majors are, for progress here will ultimately determine whether or not we can strengthen the K-16 cycle and raise math achievement to world-class levels.

On the whole, the patterns described in this report are worrisome. In both K-12 and higher education, the mathematics skills of our graduates are weak. Fortunately, however, there are exceptions from which we can draw both hope and information. You will hear about places that defy the trends and are being highly successful at raising math achievement and attainment, including a short piece by Lynn Steen discussing the efforts at St. Olaf’s College to produce high numbers of math majors.

We are again grateful to the Pew Charitable Trusts for their support of Thinking K-16.

Kati Haycock
Director
In 1989, after a thorough examination of the state of mathematics education in America, a prestigious panel assembled by the National Academy of Sciences issued a dire warning to the American people. “We are at risk,” they said, “of becoming a nation divided both economically and racially by knowledge of mathematics.”

One year later, similar concerns were echoed by the nation’s governors. When they joined with President George H. W. Bush to establish a set of national education goals, they elevated the improvement of mathematics and science achievement to a position of special prominence:

By the year 2000, United States students will be first in the world in mathematics and science achievement.

Since that time, the American people have been buoyed by reports of progress. Indeed, in the midst of flat or declining scores in many other subjects, mathematics would seem to stand out.

- According to the National Assessment of Educational Progress, for example, students at every level—elementary, middle, and high school—and of every racial and ethnic group know more mathematics today than did their counterparts in the 1970s.

- There have been gains, too, on the SAT taken by college-bound students, with scores on the quantitative portion at their highest point in 28 years.

Does this mean that we are ready, as some in Washington D.C. would have us believe, to declare victory and go on to the next subject? Hardly. As the analysis on the following pages will make clear, despite these gains, only about one in four American elementary and middle school students is proficient in math. Among our high school seniors, the number is just one in six. Within this overall mediocre performance, huge gaps persist between groups of students that continue to leave the children of low-income, African American and Latino families behind their peers. Neither have we kept pace with our international competitors who lead us in math achievement. When you add to all this a precipitous decline in mathematics degrees in higher education and a shrinking supply of math teachers, it would seem that we are in many ways in a weaker position at the beginning of the new century than we were a decade ago.

Arresting this pattern will require a coordinated, concerted attack by both higher education and K-12. For mathematics, perhaps more so than any other subject area, shows just how inextricably these two systems are linked.

Fortunately, there are some pretty powerful images out there—schools, colleges, even whole states getting much better results. And they are getting them by using strategies that aren’t beyond the rest of us.

**PART ONE: STUDENT LEARNING**

**Achievement in K-12**

The governors and president of the United States were particularly direct in setting forth their goals in mathematics and science: we wanted to be first in the world. At the time, our policymakers were nervous about the nation’s economic standing—once the unquestioned leader in the world, but by 1989, facing unfamiliar competition, particularly from the Germans and Japanese. Our leaders were convinced that maintaining our edge in the world market was linked directly to our ability to improve education.
TIMSS
Unfortunately, our students’ performance in international assessments during the 1990s did little to ease the governors’ concerns. The Third International Mathematics and Science Study (TIMSS) was conducted in 1995. Although U.S. fourth graders were not top-scorers, they put in a respectable performance that was above the international average. Yet the results for our eighth-graders slid below the average. By twelfth grade, American students were near the bottom of international rankings. [chart 1] This sad performance was true of even the top 10-15% of our students, who were bested by their peers in 11 participating countries and outperformed none.6[chart 2]

And the one ray of hope from these results—the relatively strong performance of our fourth graders—was dashed when those students were reassessed as eighth graders in 1999. At the end of middle school, the previously high-performing fourth graders ranked no higher than eighth-graders did in 1995.7

Clearly, American students were not keeping pace with their peers in other countries. This is not to say that our kids made no gains in math. Indeed by many measures they did. But the data suggest that while our students were improving, their counterparts were progressing even further, particularly in middle and high school.

NAEP
In contrast to the TIMSS results, the domestic story in mathematics achievement offered some encouraging news for the governors. Over the 1990s, results on the National Assessment of Educational Progress (NAEP) in mathematics—the so-called Nation’s Report Card—have significantly improved. At every grade level tested and for every group, student performance at the close of the decade was stronger than it was at the beginning.

Recently, some observers, including the Brookings Institution’s Tom Loveless, have raised some doubts about the magnitude of those gains. NAEP is actually two tests. The first is a long-term trends assessment that has been administered nationally since 1972 and reflects the content common to math education at that time. Unlike NAEP trends, the other test, known as the “main” NAEP, produces both national and state-level data and encompasses a broader range of mathematics, notably in the areas of statistics and mathematical problem solving. Consequently the results vary somewhat. For example, over the 1990s, students at the end of high school posted a much-discussed 7 point gain on the main NAEP.

Less known is that they showed only 3 points growth on the more traditional NAEP trends
assessment. But that both indices are up by a statistically significant amount is not in dispute.

The largest gains were made by fourth-graders. On the main NAEP they improved by 15 scale points on average, which translates into roughly one and a half years worth of learning. Eighth-graders also raised their performance an admirable 12 points. Although it pales in comparison, the 7 point net gain by twelfth-graders is still significant progress.

But there are two more worrisome features of the high school results. First, after steady improvement through 1996, high school performance declined between 1996 and 2000 by a statistically significant 3 points. Second, as a look at growth between grades four, eight and twelve makes clear, the earlier improvement in grade twelve performance was primarily a function of improvement in the elementary years. In the latter part of the decade, American high schools were apparently receiving better prepared students in mathematics, but responded by adding less value to students’ ultimate level of achievement.

How proficient are U.S. students?

Despite these gains, American students aren’t progressing far enough. The problem is clear as early as the fourth grade. In the country as a whole, only 26% of students at this grade level perform at or above proficient with an additional 43% scoring in the basic range.

Put in real terms, fourth grade students who are proficient in mathematics can:

- Find the product of several numbers when one of them is zero;
- Solve a ratio problem involving pints; and
- Draw bars on a graph to represent a situation.

Fourth-graders scoring in the basic range cannot do the above tasks. But they can:

- Use a ruler to find the total length of three line segments;
- Solve a problem involving even and odd numbers; and
- Given certain coins, show how a given amount of money can be made.

But 31%—nearly one third—of all fourth-graders perform at the below basic level, indicating that they cannot even perform this relatively straightforward mathematics. These numbers are even more worrisome when disaggregated by race. While 20% of White fourth-graders fall below basic, more than half of our African American and Latino youngsters have not been taught to the basic level.

The number of eighth graders at or above proficient is 27%. At this grade level, mathematically proficient students can:

- Use proportional reasoning to find the distance between two towns;
- Find the area of a figure; and
- Draw a line of symmetry for each of two figures.

At the other end of the spectrum, though, a full 34% of eighth graders scored below the basic level. These students cannot even solve a basic
percent problem or determine how much change a person will get back from a purchase. Again, there are significant differences among students of different races, with about one quarter of all Asian and white students performing below basic, compared to 68% of African Americans and 59% of Latinos.

By the end of high school, students who are proficient in mathematics can successfully:

- Solve a system of equations for $x$ and $y$;
- Determine the average scores given a frequency distribution of scores; and
- Analyze and explain a situation involving percent.

It’s a reasonable set of skills at this age. But only one in six American seniors can perform them. Twice as many young people—35%—are leaving high school without even meeting the basic level of mathematical knowledge and skills. While many of these students have mastered basic computation, they cannot find the perimeter of a figure; their conceptual understanding of mathematics is limited; and they are unable to consistently see mathematical relationships.\(^\text{11}\)

It’s alarming enough that 26% of White students and 20% of Asians fall below basic at the end of high school. The fact that 69% of African American youth and 56% of Latinos are in this category is potentially devastating for them and for our country.

### Gaps Between Groups

During the 1970s and 1980s, we made considerable progress in raising the mathematics achievement of minority and poor students. Between 1973 and 1986, for example, the black/white gap among eighth graders declined by about half (from 46 points to 25 points); among Latinos, there was a similar reduction (from about 35 points to 20). But African American and Latino students didn’t share equally in the gains during the 1990s and the gap subsequently grew wider, up to 32 points for African American and 24 points for Latino youth.\(^\text{12}\)[chart 7]

The patterns were essentially the same at fourth
and eighth grade: generally, some progress in both raising achievement and narrowing gaps during the 1970s and 1980s, and stable or widening gaps during the 1990s.

At the turn of the century, the data tell a very stark story. African American and Latino students in twelfth grade have skills in mathematics that are identical to White students in eighth grade.

It doesn’t have to be—indeed must not be—this way. Once again, international comparisons are instructive. The Paris-based Organization for Economic Co-operation and Development recently published the results of a test administered to 250,000 15-year-olds in 32 countries. The OECD Programme for International Assessment (PISA) was designed to find out what students know about reading, mathematics and science as they approach the end of compulsory schooling. As we saw with the TIMSS results, American students put in a lackluster performance with a rank of 19th among participating countries and a mean score just below the international average. The U.S. gap ranked an embarrassing sixth among OECD nations. Americans often blame these gaps, of course, upon gaps in socio-economic factors such as parent education. However, in some countries—including Finland, Japan, Korea, Sweden, Italy and Canada—math results shared only a weak relationship to SES. The same phenomenon is seen in the TIMSS results. Students in several nations who score high on TIMSS had far fewer of the resources at home that Americans typically credit for producing high achievement (e.g., well-educated parents, books in the home, computers). Nevertheless, they managed to perform significantly higher that their American counterparts.

**Achievement in Higher Education**

K-12 education produces considerable data on student learning. In contrast, America’s institutions of higher education have successfully fended off almost every effort to define—much less measure—what college students should be learning. Though available data are scant, the little data we have suggest that the mathematics knowledge of college students is surprisingly thin.

A large-scale study of adult literacy in 1992, for example, suggested shockingly low mathematical literacy among graduates of four-year colleges.
Nearly one in six college graduates fell at quantitative literacy level 1 or 2, which makes them functionally unable even to calculate the difference between regular and sales prices in an advertisement or use a flight schedule to make travel plans. A full half of those with bachelor’s degrees fell at or below quantitative literacy level 3. At this level, graduates are generally unable to determine shipping and total costs on a catalog order form or calculate the difference in time to complete a race based on information in a news article.\textsuperscript{15}

Perhaps not surprisingly given high school performance levels, there are stark racial differences in the mathematics skills of college graduates of different racial and ethnic groups. In fact, the gaps widen with each level of educational attainment, suggesting that higher education may be paying even less attention to educational inequities than its K-12 counterparts.\textsuperscript{16} [chart 9]

**International Comparisons**

Although the numbers are not always directly comparable, it would appear that graduates of so-called “tertiary” education abroad have significantly better mathematics skills than their American counterparts. To those who argue that this is probably because so many more American young people go to college, we would say two things. First, while it is true that we once led the world in college attendance, the U.S. now ranks 13th. Second, and perhaps more to the point, we doubt that higher attendance must necessarily produce such divergent outcomes. And indeed the relatively stronger performance of our college graduates in reading and writing (5th internationally) would seem to prove that point.\textsuperscript{17} [chart 10]
PART TWO: THE MATHEMATICS CURRICULUM

Math and science education will be strengthened throughout the system, especially in the early grades.

National Education Goal 5

In establishing their educational goals, the nation’s governors and President weren’t so naïve as to think they could get better results in mathematics without improving mathematics education. So they made a commitment here, as well.

The K-12 Math Curriculum

The 1990s saw significant increases in the numbers of students taking college preparatory mathematics courses. At the elementary level, students were doing more homework, working with more data, and spending more time on non routine problems—all of which relate to the performance gains.

Even so, other indicators do not provide much cause for celebration. International studies of mathematics curricula, for example, show that compared to other countries, the U.S. covers many more math topics, but teaches them in much less depth. Moreover, the typical American curriculum tends to repeat topics year after year. The results of the “mile wide, inch deep” curriculum speak for themselves in our international standings (see sidebar, “A Mile Wide and an Inch Deep,” page 10).

While standard setting and curriculum alignment could have been a mechanism for streamlining the curriculum to be more focused and more coherent, that mostly didn’t happen. Textbooks continue to be far too long, so long in fact that few teachers are able to get through all of them. Teachers are left to decide what is important to teach and what to leave out. This is especially problematic for elementary and many middle-school teachers who often have a weak background in mathematics.

The Math Our Eighth-Graders Learn

TIMSS data are richest on eighth grade curriculum because of analyses conducted as a part of the 1999 administration. A cross-country analysis of mathematics instruction suggests that American lessons are taught at a much lower level than those in either Japan or Germany. Researchers rated a full 87% of the lessons in American eighth grade classrooms as low in mathematical content, with the remaining 13% rated medium. None were rated high. By contrast, only 13% of Japanese lessons were rated low in content, while 57% were rated medium and 30% high in content. Germany fell in between, with 40% low in content and 23% with high content.18

That same group of international analysts rated the overall level of eighth grade mathematics lessons in the U.S. at the international equivalent of grade 7.4. In comparison, Japanese lessons averaged grade 9.1, while German lessons averaged 8.7.19

These differences were apparent in the TIMSS analyses of mathematics textbooks, as well. By grade 8, algebra constitutes 40% of the content.
A Mile Wide and an Inch Deep

Compared to the mathematics taught in other countries, the typical mathematics curriculum in the U.S. lacks coherence, is repetitive, and is more noteworthy for the quantity of topics than the quality of content. The result is a mathematics education that is “a mile wide and an inch deep.”

Such are the conclusions of researchers seeking to explain the mediocre performance of American eighth-graders in the Third International Mathematics and Science Study. While they are careful to point out that curriculum is not the only factor in student learning, what is taught and how it’s taught are nonetheless essential pieces. In the U.S., the rush to cover a wide array of math topics does not serve our students well because they rarely have the opportunity to develop a deep understanding of key mathematical concepts. Our young people are “taught” more topics than their peers in high-achieving nations, but their disappointing performance shows that in this case, more is clearly less.

William H. Schmidt and his colleagues found that the middle school curriculum, in particular, “is replete with repetitious and non-challenging material.” For example, the American middle school curriculum covers between 27 and 32 topics each year compared to the international average of 21 to 23. Our students also see the same concepts taught year after year—math topics stay in the U.S. curriculum an average of two years longer than the international average—but apparently with little or no added depth.

Mathematics textbooks are as much a cause of the “mile wide, inch deep” phenomenon as a symptom. Driven to accommodate 50 different state markets, textbook publishers have hedged their bets by including as many topics as possible in order to meet the various state standards and curriculum requirements. The resulting bulk of these textbooks is well known to our youngsters who labor under the weight in their backpacks. An examination of fourth- and eighth-grade textbooks showed the U.S. math books covered a whopping 30 to 35 topics on average. In comparison, their counterparts in Germany addressed 20 topics and Japanese texts focus on just 10. Both are countries that outscore the U.S. And our competitors manage to do more with smaller books. The topics that are addressed in their books have more pages devoted to them than American textbooks offer for even the most important math concepts.

Between voluminous state math standards and the backbreaking tomes disguised as textbooks, the tools we give teachers to teach mathematics conspire against the kind of focused, in-depth instruction our students need to become mathematically proficient. The TIMSS survey of teachers confirms that indeed teachers are covering many more mathematical topics than the international average. American teachers reported that they teach 93% of the TIMSS content areas to their eighth-graders—way beyond the international average of 75%. In addition, the instruction is largely textbook-based. U.S. students are far more likely than their peers to work on textbook problems or worksheets.

In another report, researchers Schmidt, McKnight and Raisen wrote:

Both conventional wisdom and a considerable body of research ... suggest that focus and selection are needed in situations in which too much is included to be covered well. This impact of these unfocused curricula and textbooks in mathematics and science likely includes lower “yields” from mathematics and science education in the U.S.

But until the U.S. manages to produce a mathematics curriculum that is at least as deep as it is wide, American students are not likely to produce the “yields” policymakers say they want and our young people need to be successful.


of Japanese mathematics textbooks and 25% of German math texts. But in the U.S., algebra makes up only 10% of eighth grade mathematics books.\(^{20}\)

**AAAS Textbook Reviews**

Frankly, though, it is unnecessary to go abroad to find criticism of American mathematics textbooks. It turns out to be quite plentiful right here in the U.S.

The American Association for the Advancement of Science, for example, has conducted detailed reviews of textbooks in both mathematics and science. Their reviewers have found mathematics textbooks particularly lacking in effective instructional strategies, especially for helping students understand algebra. According to George Nelson, then-Project 2061 director, “We do see solid improvement in some of the newer materials, but each book had major shortcomings. Unfortunately, the areas where most books are the weakest are those that are most critical in helping all students achieve, such a building on the knowledge that students may already have and dealing with their misconceptions.”\(^{21}\)

While the reviewers gave three of the twelve textbooks they reviewed high ratings, these were not among the best sellers. The most widely used middle-school texts were all found to be unsatisfactory. When one considers the dominant role textbooks typically play in math instruction, the use of inadequate texts is worrisome.

**Completions of College Preparatory Mathematics Courses**

In 1983, the National Commission on Excellence in Education called national attention to a “cafeteria style” high school curriculum, wherein “desserts could easily be confused for main courses.” Since that time, there have been significant improvements in the number of high school students completing college preparatory mathematics courses. (Indeed, the numbers are so impressive that one is hard pressed to explain why they haven’t led to higher levels of student achievement.)

Since 1982, the proportion of graduates completing Geometry, for example, has grown from 47% to 75% in 1998. At the same time, the number of students completing Algebra II, the minimum content typically required in order to enroll in college-level mathematics, has grown from 40% to 62%. There have been significant increases in Pre-calculus and Calculus, as well. The number of high school graduates who took Pre-calculus is up from only 6% two decades ago to 22% today. Over the same time period, the number in Calculus doubled from 5% to 10%.\(^{22}\)

Despite the overall sweep toward college preparatory coursework, minority youngsters continue to be underrepresented in such courses. While about two-thirds of Whites and Asians take Algebra II, for example, only about half of Latino, African American and Native Americans take this course. [chart 12] Differences are even larger for Pre-Calculus, with 41% of Asians and 25% of Whites taking this course, compared to about 15% of African American, Latino and Native American students.\(^{23}\)

**Mathematics Education at the Higher Education Level**

In 1991, a prestigious panel of mathematicians was convened by the National Research Council to review of mathematics education at the collegiate level. Their conclusion: “The profile of mathematics in higher education is not that much different from that of mathematics in high school.”\(^{24}\)

And indeed, the data they compiled were rather surprising.

- Two-thirds of all college mathematics enrollments were below the level of calculus.\(^{25}\)
- 96% of all college mathematics enrollments
Growing Math Majors at St. Olaf College

Lynn Arthur Steen

The mystery of mathematics is how it can be so shunned at the moment of its greatest triumphs. Behind the decoding of the human genome one finds algorithms of patterns and combinations; behind the animation of Oscar-winning films one finds algorithms of geometry and projections; behind the new on-line financial markets one finds algorithms of data mining and secure communication. No longer confined to physics and engineering, mathematics is the foundation for what is often called the “new economy.”

Yet for years headlines have proclaimed depressing news about the poor standing of U.S. students in national and international assessments. Academics worry about a parallel trend: the percentage of U.S. college graduates who major in mathematics has declined steadily for three decades, and is now only one-third of what it was in 1970. Only one in a hundred college graduates now majors in what used to be called the queen of the sciences, and only one in 25 of those majors goes on to a Ph.D. in the mathematical sciences. ¹ And this at a time when advanced mathematical tools are being employed more widely than ever before, and when mathematicians command six figure salaries in enterprises ranging from Wall Street to Hollywood.

In this context it is not surprising that people are intrigued by the mathematics program at St. Olaf College which, for all but two of the last twenty years, has produced mathematics majors at more than eight times the national average. These majors are no slouches: they have gone on to earn Ph.D degrees in the mathematical sciences at a rate 50% higher than the national average. As a consequence, when the National Science Foundation reported on the undergraduate origins of mathematics Ph.D. degrees earned between 1990 and 1995,² St. Olaf ranked sixth in the nation, behind only UC-Berkeley, Harvard, MIT, Chicago, and Cal Tech.

It is natural to wonder how this came to be and whether the St. Olaf “secret,” if there is one, might be adopted by other colleges. St. Olaf is a liberal arts college of 3000 students, half from Minnesota. These students are pretty much like students in other moderately selective liberal arts colleges with SAT scores mostly between 1100 and 1300. Our faculty work about as hard as their peers in other colleges, although based on national norms they are all underpaid. As an institution, St. Olaf ranks just below 50th in the US News college rankings of national liberal arts colleges, and is better known for music than for mathematics in its mostly upper mid-west constituency. To produce that many mathematics majors, we mostly have to grow our own.

In the early 1970s, as mathematics departments everywhere struggled to adapt a curriculum that had become too rigid for changing student interests, St. Olaf mathematicians decided to let students help shape their own major by “contracting” with their departmental advisors for a selection of advanced courses that both met their career interests and contained an appropriate breadth of mathematics. In effect, the faculty joined their students as partners in the struggle to modernize the curriculum as mathematics moved hesitatingly into the computer age. In 1981 a committee of the Mathematical Association of America (MAA) cited St. Olaf’s “contract major” as a major source of its success.³

Fifteen years later, following a decade in which the percentage of St. Olaf mathematics majors remained consistently in double-digits, another visiting team from the MAA identified a variety of additional features that helped explain the department’s success.⁴ Chief among these is its continuous emphasis on students as the focus of the department, from a long-standing placement program that typically achieves a 90% success rate in calculus⁵, to numerous extra-curricular activities (a student MAA chapter, weekly newsletter and colloquium, problem-solving groups) and social events (ice cream socials, annual pig roast, and a math department music recital). These show students “that mathematics is not an elite discipline for the few, but a lively and accessible subject appropriate for all.”

As the MAA reviewers noted, these student-oriented activities sustain but do not alone create the environment that attracts well over 100 upper class mathematics majors. The environment, they suggest, depends
on a faculty that is never static, that is never satisfied with the status quo, that is always looking for ways to improve and for grants to support these ideas. The innovative contract major was one such effort; others include:

- undergraduate research opportunities for advanced mathematics students dating back more than thirty years;
- early and extensive integration of statistics, computing, and computer science;
- a January-term “practicum” for team work on industrial or government problems (e.g., streamlining Northwest Airlines security lines);
- leadership in national problem-solving competitions;
- active participation in professional societies—which has led several faculty to editorship and officer positions;
- a Visiting Master Teacher program for outstanding high school teachers;
- several external grants for curriculum and computer laboratory development;
- a grant-supported program of “teaching post-docs” for new Ph.D.s to gain experience in liberal arts college teaching;
- special mathematics programs in Budapest and (recently) at the Biosphere in Arizona.

These faculty-instigated programs help show students that mathematics is an active field—not something that exists only in textbooks and exams. The enthusiasm of faculty for their work rubs off on students who see the department as a fun place to hang out and mathematics as a subject with varied and interesting opportunities for careers. Quite a few go into high school teaching, and some of their students subsequently attend St. Olaf primed with stories about what it is like be a St. Olaf math major.

Each layer of explanation, like a Russian doll, reveals yet another question: What sustains and energizes the faculty? The answer, in part, is a written departmental consensus, regular revised, on broad standards for the integration of faculty teaching and professional activity. This statement, similar in spirit to the “reconsidered” scholarship advocated by Ernest Boyer, is anchored in a belief that mathematics is an active, hands-on, “big tent” subject, a true liberal art offering intellectual habits of mind that are as useful in the film studio as in the genomics lab.

Faculty professional work is as varied as student interests, and as often as not is done with some degree of collaboration. In the end, the true energizer of faculty is interactions with students.

Lynn Arthur Steen is Professor of Mathematics at St. Olaf College.


were in courses that are also taught in high schools.\textsuperscript{26}

Enrollments in remedial mathematics continue to be alarming. Nearly one in four college freshmen takes a high school level mathematics course. In two-year colleges and high-minority institutions the number is about one in three.\textsuperscript{27} [chart 13] Even at highly-selective, top-rated doctoral granting institutions today, about 20\% of total student enrollments in mathematics are at the remedial or pre-calculus level.\textsuperscript{28}

The NRC panel went on to note what for them was a most disturbing pattern. “Every year, from high school through college, one in two students studying mathematics stops.” In other words, it’s not as if students are simply entering college with weak skills, burnishing those skills and moving on. Rather, in most cases, they seem to be completing the required minimum and then fleeing.

**PART THREE: TEACHERS**

The number of teachers with a substantive background in math and science will increase by 50\%.

**National Education Goal 5**

**Teaching Mathematics in K-12**

When setting national education goals, governors knew intuitively what research has now confirmed: that teachers with a strong background in mathematics are more effective in producing student achievement. When they set those goals, large numbers of American students were being taught mathematics by teachers whose background in the subject was tentative at best. In 1988, about one in three high school math students was taught by a teacher who lacked a major in either mathematics, math education or related field (e.g., engineering). Twelve years later, these numbers are unchanged.\textsuperscript{29}

There are even bigger problems in middle school, where 61\% of our students are currently taught mathematics by teachers who did not themselves study enough mathematics to earn even a minor in math, math ed or related fields.

But there are also different patterns for different types of students.\textsuperscript{30}
• For example, math courses in schools with high concentrations of minority students are more likely to be taught by teachers without a background in mathematics. Indeed, in math courses in high schools with large concentrations of minority students, 32% of the teachers lacked even a minor in the subject area compared to 23% in mostly White schools.

• The same pattern holds for high schools serving low-income families, which are considerably more likely than more affluent schools to have math classes taught by underqualified teachers.31[chart 14]

• Inequities are even greater at the middle school level where 70% of all math classes in high-minority schools are taught by teachers who lack even a minor in mathematics or related field compared to 55% of math classes in mostly White schools. [chart 15]

Students in other countries are far more likely than American youngsters to be taught mathematics by teachers who majored in mathematics. Across the countries participating in TIMSS, an average 71% of eighth-grade math teachers were mathematics majors in college; in the U.S, only 41% had majored in mathematics.32[chart 16]

In the 1990s, the average education major completed only 6.3 units—or about two semester-long courses—in mathematics.33 These may not even have been college-level mathematics courses. Indeed, according to a U.S. Department of Education report, education majors were more likely than other undergraduates to have taken remedial mathematics while enrolled in college.34

Despite this, studies of the mathematical literacy of adults suggest that the mathematics skills of teachers are not significantly different from those of other college graduates—perhaps because many of them have had to pass a basic mathematics assessment to get their licenses in the first place. Yet there is little comfort here, because college graduates in general turn out to have surprisingly weak mathematics skills. Nearly half of all col-
college graduates—and, yes, nearly half of all teachers—fall at quantitative literacy level 3 or below. As discussed earlier, this means they can compute correct change but they still can’t perform multi-step problems even if the math operations are fairly basic.

**Teaching Mathematics in Higher Education**

Higher education has its own issues related to teaching mathematics. According to the National Research Council, enrollment in undergraduate mathematics courses in the 1980s rose more than twice as fast as did faculty resources, pressing the mathematics community to respond to these demands in the least expensive ways. Large introductory courses became the norm, as did the extensive use of graduate assistants and part-time faculty. In research universities, teaching assistants and part-time faculty teach nearly 90% of all remedial mathematics courses; in other colleges, such faculty teach over 60% of all remedial mathematics.

Much like the pattern in K-12, high-end students are far more likely to be taught by more qualified teachers—in this case, full-time faculty members—than are students who enter at lower levels. In freshman-level courses at research universities, for example, graduate students and part-time faculty members teach 54% of the remedial mathematics courses, 49% of pre-calculus courses, and only 19% of calculus courses.

WHERE ARE THE MATH MAJORS?

By the year 2000, the number of U.S. undergraduate and graduate students, especially women and minorities, who complete degrees in math, science and engineering will increase significantly.

**National Education Goal 5**

At the beginning of the 1990s, there were more than 73,000 upper division mathematics majors in U.S. colleges and universities. By 1999, that number had shrunk to approximately 56,000. Degree production has dropped as well: In 1991, approximately 15,300 bachelor’s degrees were granted in mathematics. By 2000, the number of mathematics baccalaureates dropped to 12,100, and represents a mere 1% of total bachelor’s degrees, down from 1.4% nine years ago.

In truth, though, the decline in mathematics degrees began well before the national goals were set. In 1971, for instance, U.S. colleges and universities granted approximately 25,000 bachelor’s degrees in mathematics—3% of all bachelor’s degrees awarded that year. The decline has continued to this day.
Putting Math Standards to Work

For many years, California has been blessed with a set of content-rich professional development initiatives that are the envy of many other states. Collectively known as the California Subject Matter Projects, these truly K-16 initiatives include the 22-site California Mathematics Project, as well as similar projects in other disciplines.

Administered by the University of California system, the subject matter projects are based on college campuses throughout the state. Their goals are to provide standards-based professional development, engage university faculty in collaborative work with schools and districts, and nurture discipline-based professional communities.

Like many other university-based professional development programs, the California Mathematics Project has, since its inception in 1983, provided rigorous and engaging learning opportunities to teachers throughout the state. Participating teachers consistently rated their experiences as among the best of their careers.

But, again like many other higher education-based professional development programs, the CMP didn’t always attract teachers from the states neediest schools, it didn’t always focus on what the state thought was important, and it didn’t measure its impact on teacher and student learning. Several years ago, however, all of that changed rather abruptly, when the State of California demanded a refocusing of these precious resources.

Now, project resources are much more tightly focused on schools with high poverty, large numbers of beginning teachers and low student achievement. And the work is also directly focused on helping teachers to teach state standards.

The process of focusing professional development on the state mathematics standards proved initially to be quite difficult. California, as in many states, has long lists of standards. Indeed, there are 25 separate standards for Algebra 1 alone. In order to make the standards more useful, CMP leaders gathered a committee of prestigious mathematicians and teachers to prioritize the statements as “important,” “very important,” and “very, VERY important.” Using this framework, the committee was able to reorganize the state’s voluminous learning objectives around a manageable number of main ideas—so-called “power standards.”

These power standards, in turn, formed the nucleus of a new set of California Professional Development Institutes. Teachers participating in these institutes were assessed on their own knowledge in these core areas, so that institute leaders knew where they were starting. Teachers then focused their time at the institutes—between 40 and 120 hours each—on these concepts. They received “maps” of where these core concepts were covered in the textbooks they were using. And they also received well-designed “replacement units” that they could use with their students.

Above all, they spent a lot of time deepening their own content and pedagogical knowledge in areas directly relevant to what they needed to teach their students. Many of the teachers entered the project with weak content knowledge. End-of-institute assessments showed real growth, but also that more help is needed.

Because teachers participating in the project receive 80 more hours of follow-up during the year, California has a mechanism to help those teachers keep growing. But as they ponder the results of the pre- and post-teacher testing and of the assessments of these teachers’ students that are yet to come, project leaders are thinking hard about how to provide even more.
We are also awarding fewer graduate degrees in mathematics, although the decline is not nearly as precipitous as that in undergraduate degrees. In 1970, U.S. colleges and universities awarded about 5,700 master’s degrees in mathematics; in 2000, the number was about 3,400.\(^{40}\) Over the same time period, the number of doctoral degrees awarded in mathematics has stayed somewhat stable. However, the discipline’s share of all doctorates awarded has declined by half to a current 2%.\(^{41}\) [chart 18]

Bachelor’s degrees in mathematics are awarded mostly to U.S. citizens (96% of all mathematics bachelors degrees), but graduate degrees often go to foreign students. In 1999, foreign students earned 31% of all master’s degrees in mathematics and 47% of all doctoral degrees. Note, by contrast, the proportion of degrees going to African-American and Latino students. Together, these groups accounted for 13% of the bachelor’s degrees, 7% of the master’s degrees, and just 2% of the doctoral degrees in mathematics.\(^{42}\)

As U.S. universities were reducing their output of mathematics degrees, universities in other countries were increasing them, often dramatically. Between the years 1985 and 1999, the proportion of bachelor’s degrees in mathematics and computer sciences increased from 2 to 5% in Germany, from 1 to 4% in Spain, and from 4 to 7% in Ireland. In contrast, the proportion of such degrees in the U.S. dropped from 6 to 3% of all bachelor’s degrees.\(^{43}\) [chart 19]

**Teachers: A Huge Supply Problem**

According to a 2000 report from the National Commission on Mathematics and Science Teaching, we will need to hire about 240,000 math and science teachers during the coming decade.\(^{44}\) If we assume that half of these positions are in mathematics that would translate into the need for 120,000 math teachers. But if we also assume that present graduation trends continue, we will award bachelor’s degrees to about 120,000 mathematics majors over the same time period. Thus, every mathematics graduate would have to go into teaching in order to meet the demand for math teachers.

But most mathematics graduates do not, of course, go on to become teachers. Indeed, in the 1990s, only 29% of recent graduates with a bachelor’s degree in mathematical sciences went into teaching in either K-12 or post secondary.\(^{45}\) Salaries don’t provide much incentive for them either. The average starting salary for mathematics majors with bachelor’s degrees is $46,466.\(^{46}\)
comparison, beginning teachers earn an average $27,989 per year. The average annual salary for teachers overall is $41,820—still less than starting salaries for math majors in nonacademic fields.47

WAYS OUT

It doesn’t take a mathematics major to complete this mathematics problem. If present patterns continue, there is literally no way to raise mathematics achievement to where it needs to be at every level. It is very difficult to produce higher mathematics achievement in K-12 without teachers who have a deep and flexible knowledge of mathematics. But higher education’s output of graduates with a strong foundation in mathematics is falling precipitously. Even what is arguably the strongest part of our mathematics education system—our doctoral level programs—may not be able to withstand the challenge inherent in the combination of a post September 11 chill in non-resident student enrollments and increased competition from foreign universities.

If we just wanted to prove our case that our two systems of education—higher education and K-12—are so deeply intertwined that you can’t change one without also changing the way the other does business, we would simply rest here. But proving that case is not our aim here. Rather, we want to stimulate concerted, K-16 action by providing not just a rundown of the problem, but also at least a few concrete images of how we might solve it.

Fortunately, while the national data are pretty bleak, when you poke around underneath the data, you see quite quickly that some institutions are doing a much better job than others at getting results. For example, St. Olaf’s College in Minnesota bucks the trend seen in other institutions by gaining students as math majors each year as opposed to losing them (see sidebar by Lynn Steen for a full description, page 12 ). Through the UTeach initiative, the University of Texas, Austin is highly successful at recruiting talented math and science students into teaching, even though other careers might be more lucrative.

K-12 education has its success stories, too. Several states are moving to raise their expectations for all students by establishing college preparatory courses, including mathematics, as the default curriculum. This is an essential first step. But while we know that enrolling in high level courses improves achievement, we aren’t going to get the results we need unless we make sure that every student has the benefit of a competent teacher.

The California Math Project provides a powerful example of a large-scale professional development initiative to help current teachers improve their effectiveness. Through collaborations between K-12 and state universities, teachers are provided with in-depth, discipline-based professional development focused specifically on content and instructional strategies for teaching mathematics (see sidebar, “Putting Math Standards to Work,” page 17).

Also, schools in Pittsburgh are seeing a big payoff as a result of the professional development provided by the Learning Research and Development Center. Students attending schools that have fully engaged in the LRDC support strategies are making the most gains in achievement, and the achievement gap between white and African American students is closing. In these so-called “strong implementation” schools, African American students are outperforming even the white students in “weak implementation” schools. Moreover, the gap in math skills has been wiped out.48

The status of mathematics achievement in the U.S. still leaves us at risk. We can take some encouragement from the gains in K-12 over the past decade, particularly among elementary students. But as we have shown in this report, the gains don’t go far enough. We have done too
little to close the achievement gaps that still keep too many low-income students and students of color so far behind. And we are losing ground in higher education, once our greatest source of national pride.

Clearly our efforts to date have not been of a scale commensurate with the magnitude of this problem. But it is not too late if more communities and states will take the kinds of steps below—bold steps extracted from some of the best work across the nation.

**ACTIONS FOR CHANGE**

In the end, the solutions—as well as the will to stick with them—must come from mathematicians themselves. Nonetheless, there are many things policymakers, K-12 and higher education leaders can do to support change.

**Bring Them Together**

Accordingly, our first and most fervent recommendation—to both statewide and local leaders—is to start by engaging mathematicians of every sort: K-12 teachers, higher education faculty in mathematics and mathematics education, faculty from other disciplines that use mathematics, and employers in math-based fields. Bring them together. Show them the data. Encourage them—together—to tackle the myriad problems that are so clear in the data. Invite them, in other words, to take responsibility for improving teaching and learning in mathematics from kindergarten through college.

**Pull together the data for your state or community.**

The data in this publication can be a good conversation starter for these meetings. A PowerPoint version of the story told on these pages is also available on our website. Both our publications and PowerPoint presentations can be downloaded and distributed at no cost. Please feel free use them in your meetings.

But national data are no substitute for the data for your state or community. At the very least, you will want to pull together data on how the mathematics pipeline looks in your region. These data can serve, later on, as the foundation upon which goals can be set and progress measured.

**Some Ideas for Action Worth Considering**

We believe that the K-16 conversations with your mathematics professional will yield ideas that are best suited for your states and communities. But we share some worthwhile ideas that we’ve stumbled on, either in the research or in schools or colleges across the country.

**Assistance For Current Teachers**

1. **Identify the MOST important standards.**

Bill Schmidt is not the first to identify the “mile wide, inch deep” character of the American mathematics curriculum. This problem has been widely acknowledged for years. The standard-setting process could, of course, have been a powerful tool in narrowing and deepening the curriculum. Unfortunately, however, in mathematics as in most other subjects, the standard-setters in many states may have gotten carried away. In the interest of gaining consensus, some states put literally everything imaginable in their standards.

Given the politics of the process, though, most states are not about to revisit their standards—at least not now. So the question is what to do. The process, such as that adopted by the California Math Project, certainly provides one possible answer: get eminent mathematicians in a room, and ask them to divide the standards into “important,” “very important,” and “very, VERY important.” One way to help sift through the standards would be to get clearer on the mathematics knowledge and skills that are actually necessary to succeed in college in (1) math-based fields; and (2) non-math based fields.
2. Refocus Curriculum, Assessment and Professional Development on “Very Very Important” Standards

Provide every mathematics teacher with a guide that maps their textbook against “very very important” standards.

Almost no teacher can cover all of the chapters in a typical, fat American textbook. So they either go as far as they can, or hop around, making their own choices. You could help already overburdened teachers make good choices by developing maps of the textbooks most widely used in your state, showing them exactly where the most important standards are covered.

Design powerful replacement units for the “very very important” standards.

Often what’s in the textbook isn’t good enough. Accordingly, you might want to engage some of your most thoughtful teachers—from both K-12 and higher education—in designing powerful “replacement” units, with lesson plans, model student assignments, and even examples of student work that meets standard.

Focus professional development on the “very very important” standards.

A study a few years back by Cohen and Hill suggested that well designed replacement units (or curriculum “chunks”) can be a powerful learning tool for teachers if professional development is built around them. Unlike college-level mathematics courses, which seldom develop the specific mathematical knowledge that teachers need to teach the curriculum at their grade level, this kind of professional development is more focused. It provides immediately usable lessons, as well as an opportunity to deepen teacher knowledge in precisely the domains they must teach.

Help teachers reach greater consistency in the teaching of certain courses by developing some combination of common mid-stream and end-of course tests, and common “benchmark” assignments.

The Southern Regional Education Board is moving in this direction with the development of end-of-course assessments that will be available to the SREB membership. Other resources are available through the National Science Foundation, NAEP, and on state web sites.

Increasing the Numbers of Students Studying Mathematics at All Levels

In K-12

1. Make the college prep mathematics curriculum the default curriculum for all students. Research is very clear that students will learn more mathematics when they are taught a college-preparatory mathematics curriculum. And this is true for both students who think of themselves as college bound and the so-called “vocational” students. Yet we still program only about half of all students into the full college prep series.

Rather than continue to make incremental change, we would suggest that states and districts make the college prep mathematics curriculum the default curriculum for all students. Texas has already done this by state law; the California legislature has a similar bill pending. But districts don’t have to wait on the state: others can follow the route that districts like Ysleta in Texas, and San Jose Unified and New Haven Unified in California have taken by putting all students in the college prep sequence.

2. Help students before they fail. Rather than waiting until summer to provide extra help to students who fail to pass these courses, identify those likely to fail the courses during the summer BEFORE, and teach them some of the core concepts up front. Then, when they get to those concepts in the actual course, they’ll feel familiar. Uri Triesman and his colleagues on the math faculty at University of Texas, Austin have done this for years with substantial
success. High schools—even middle schools—should do the same thing.

3. **Raise the certifications requirements for middle-school mathematics teachers to include more substantial mathematical knowledge.** In many states, teachers with generic K-8 credentials can teach middle-school subjects including mathematics. The problem is that the preparation to teach elementary education is aimed at producing a generalist of all subjects. But the demands of middle-school mathematics require a teacher with a much deeper understanding of the subject.

The University System of Georgia has addressed this issue by adopting new course requirements for aspiring middle school teachers that require two academic concentrations of 12-15 units each. At least 9 credit hours in each concentration must be at the upper division level.

4. **Mine new sources to attract individuals knowledgeable in mathematics into the classroom.** Launched in 1997, the New Teacher Project works with school districts across the country to recruit and train talented, well-educated adults from other professions to fill hard-to-staff teaching positions, including secondary mathematics. In just three years, the New Teacher Project has attracted 13,000 applicants for positions in the New York City school system, of which 1,400 were qualified in mathematics. Of the eventual pool of “teaching fellows,” 22% had graduate degrees; their average college GPA was 3.5; and 42% were people of color.

5. **Enroll high school students in college courses.** Instead of accelerating high-end students through Advanced Placement, which can have perverse effects during a teacher shortage, consider accelerating them into actual college courses—taught by higher ed faculty members. Many states and districts are doing precisely this. By putting in place a funding system that doesn’t penalize K-12, the state of Colorado has secured high participation in its concurrent enrollment programs. Utah has also made a major push to get students to complete up to an Associate’s Degree while still in high school. The City University of New York also has a significant concurrent enrollment initiative, called College Now.

6. **Consider doing the same thing for low-end students.** Instead of waiting till they get to college to teach “remedial mathematics,” have the colleges teach it to high school juniors or seniors. California’s Glendale Unified School District is one district that has done this for its middle- and lower-achieving students.

**In Higher Education**

1. **Set goals for increased production of Baccalaureate degrees in mathematics.** Engage departmental faculty and students in strategizing about how to reach those goals. Look, in particular, at the St. Olaf’s experience. Examine the lessons. Reward progress generously.

2. **Engage mathematics faculties in strategizing, too, about how to produce more—and better—teachers of mathematics.** Consider creating a home-grown version of UTeach. Ask your faculty what resources they would need to produce the same kind of increases in the number of students preparing to become math teachers.
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Save the Dates

The 12th Education Trust National Conference

Grand Hyatt Hotel
Washington, DC

November 16–19, 2002